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Aspects of Higgs Inflation

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Abstract

The earliest stage in the history of the universe is successfully modelled by cosmic inflation, a period of nearly exponential expansion. Due to inflation, the universe became spatially flat, old, and statistically homogeneous with small inhomogeneities in the energy density that later acted as seeds of structure.

In the simplest scenario, inflation is driven by a scalar field, the inflaton. In the Standard Model (SM) of particle physics the Higgs boson is the only fundamental scalar field, which makes it an interesting candidate for the inflaton. However, pure SM Higgs potential does not produce the requirement amount of inflation. Instead, successful inflation can be obtained by adding a large non-minimal coupling between the Higgs and gravity which effectively flattens the potential and allows for an extended period of inflation. This is known as the Higgs inflation model.

The effective theory of non-minimally coupled Higgs and gravity is non-renormalisable and breaks perturbative unitarity at an energy step below the inflationary regime. This prevents the use of perturbative quantum field theory methods in running the couplings up to the inflationary scales. It has been proposed, however, that effects of the non-perturbative or the non-renormalisable physics below the inflationary scale could be parametrised by threshold corrections which amount to undetermined jumps in couplings of the model. This leaves basically three parameters determining the Higgs inflation potential: the jumps in the Higgs self-interaction and the top Yukawa couplings and the strength of the non-minimal coupling between the Higgs and gravity. In addition to these free parameters, the choice of the gravitational degrees of freedom, or the choice between the metric or the Palatini formulations, affects predictions of Higgs inflation.

This thesis consists of three articles investigating the robustness of Higgs inflation predictions. By varying the three aforementioned parameters both in the metric and Palatini formulations one can construct different kinds of *features* in the inflationary potential which widen the range of predictions of Higgs inflation. We also consider the combined Higgs-Starobinsky model of inflation that is motivated by quantum corrections. This analysis is performed in the metric formalism.

Detailed understanding of Higgs inflation predictions is crucial in contrasting the scenario against future observations of the Cosmic Microwave Background and gravitational waves which may favour some realisations of Higgs inflation and rule out others. This may help to understand the microscopical mechanism of inflation, and, if the Higgs really is the inflaton, also shed new light to the high energy behaviour of the SM coupled to gravity.

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List of Included Papers

This thesis is based on the following publications [1–3]:

I Observational Signatures of Higgs Inflation
V.-M. Enckell, K. Enqvist, S. Nurmi
JCAP **1607**, 047 (2016)

II Higgs Inflation at the Hilltop
V.-M. Enckell, K. Enqvist, S. Räsänen, E. Tomberg
JCAP **1806**, 005 (2018)

III Higgs-R^2 Inflation — Full Slow Roll Study at the Tree Level
V.-M. Enckell, K. Enqvist, S. Räsänen, L.-P. Wahlman
HIP-2018-37/TH

In all of the papers the authors are listed alphabetically according to particle physics convention.

The author's contribution

The author participated in writing the papers **I** and **III**. In paper **I** the author participated in both analytical calculations and numerical computations. In paper **II** the author carried out half of the numerical computations. In paper **III** the author participated in both analytical calculations and numerical computations.

Contents

Chapter 1

Introduction

The basic interactions of Nature are successfully described by two different theories of physics. The long distance phenomena accounted for by gravitation are described by general relativity [4]. A subatomic world, on the other hand, is governed by the Standard Model of particle physics [5]. Although both theories have reached an excellent agreement with observations, there are unexplained phenomena that require at least some modifications of these theories. A prime example is cosmological inflation [6–8], the nearly exponential expansion of the early universe, that might follow from a minimal extension of either of these theories.

The Standard Model of particle physics (SM) describes three basic interactions that govern the subatomic world. The latest success within the Standard Model has been the detection of the particle corresponding the scalar Higgs field, the Higgs boson. It was theoretically predicted already a half-century ago [9–11] and discovered for the first time in 2012 at CERN's Large Hadron Collider [12, 13]. The great significance of the Higgs boson comes from the spontaneous symmetry breaking that results in the SM particles becoming massive. As the only fundamental scalar field, the Higgs may also play a central role in the very early universe (see e.g. [14–17]).

Regarding the physics of large length scales, in recent years cosmological observations have transformed theoretical cosmology from qualitative to quantitative science. The Cosmological Microwave Background [18], the Hubble expansion [19], and the formation of cosmological structures [20] are prime examples of phenomena that are described in great detail by the standard big bang cosmological model. This model is known as the $\Lambda\mathrm{CDM}$ model [21]. Here Λ refers to the dark energy of the universe [22], which is needed to explain the observations on distant supernova redshifts that reveal the accelerating expansion of the present universe [19]. CDM refers to Cold Dark Matter [23, 24] which is the main component of matter in the Universe.

Despite the success of the Standard Model and general relativity, there is still a host of problems that remain unexplained. For example, although the cosmological effects of dark matter and dark energy are well understood within the ΛCDM model, the microscopical origin of these phenomena remains unknown. On top of that, the Standard Model contains problems such as unexplained neutrino masses [25] and the generation of the matter-antimatter asymmetry [26] in the universe. One central, unexplained mystery challenging both modern physics theories, is the origin of cosmological inflation [6]. It covers the rapid period of nearly exponential expansion of the early universe and took place before the start of the hot Big Bang era described by $\Lambda\mathrm{CDM}$ cosmology.

Regardless of the precise mechanism behind inflation, it explains several features of the present universe. The homogeneity, isotropy, and spatial flatness of the present universe, which would otherwise appear as boundary conditions for the spacetime, become just natural consequences of the exponential expansion of the early universe [6, 8, 27]. Moreover, inflation serves as an explanation for all structure in space by generating primordial inhomogeneities, the first seeds of structure [21, 28].

Many models have been proposed for inflation. Most often, it is considered to be driven by some energetically dominating scalar field. One could thus hope that the Higgs field, being the only fundamental scalar field of Standard Model, could have the properties required to give rise to inflation. However, this is not the case in the pure Standard Model, where the Higgs potential is too steep to yield enough inflation. The situation changes if there is a strong non-minimal coupling between the Higgs field and gravity [14]. Then, the potential of the Higgs includes also an inflationary regime at high field values. The tree-level predictions of this model also agree with CMB observations [14].

The non-minimal coupling to gravity makes the Higgs field nonrenormalisable and breaks perturbative unitarity of the model below the inflationary regime [29–31]. This prevents computation of quantum corrections over the scales between the low energy SM regime and the high energy inflationary regime. The nonrenormalisable physics at these intermediate energies can be parametrised, however, resulting in an ambiguity in the predictions of the model [29, 30]. In the research articles **I** and **II** a comprehensive study of Higgs inflation with quantum corrections included were performed for the first time.

In addition to the Higgs field, other scalar fields may be present and contribute to the inflationary dynamics. One example of such a field is the scalaron field arising from the simplest version of $F(R)$ theories of modified gravity [27, 32]. Simultaneous action of such scalaron and the Higgs fields would lead to a multified inflationary scenario, for which a comprehensive study of the cosmological signatures is performed in the research article **III**.

The thesis is organised as follows. In Chapter 1 we build the framework for the Higgs inflation model by introducing the basics results of general relativity and Standard Model Higgs. In Chapter 2 we first review the general inflationary scenario and introduce the cosmological observables. After that, Higgs inflation and Starobinsky inflation (which is driven by the scalaron field) are considered at the tree level. In Chapter 3 we focus on the radiative-corrected Higgs inflation. The quantum corrections for the Higgs, the parametrisation of the non-renormalisable physics and the possible features in the inflationary potential are introduced in detail. I review also the results for the different predictions for the Higgs inflation, based on the research articles **I** and **II**. Similarly, in Chapter 4 we first introduce the Higgs-Starobinsky model and then focus on the possible outcomes of it that are based on the research article **III**. We conclude with a discussion in Chapter 5.

Chapter 2

Spacetime and matter

In this section we consider two theories of modern physics: General Relativity (GR) which is the theory of spacetime, and the Standard Model of particle physics (SM) which is the theory of matter. In the case of GR, the Einstein-Hilbert gravity and its two possible formulations are reviewed. We also treat generalised gravity and its implications. In the case of SM, we consider the Higgs sector and two kinds of corrections to the classical picture: the radiative corrections and the temperature corrections.

2.1 General Relativity

According to General Relativity gravitation manifests itself as the curvature of four dimensional spacetime [33]. The curvature is universal in a sense that gravitational field cannot be detected by means of local experiments. This is known as an Equivalence Principle [4]. It leads to mathematical description of the spacetime as a curved manifold that locally reduces to Minkowski space. Let us next introduce the central objects and results of GR following [4].

The central objects in a curved spacetime are the metric tensor $g_{\mu\nu}(x)$ which defines the geometry of the manifold, and the connection $\Gamma^{\lambda}_{\mu\nu}$ which relates the vectors on a tangent space to the nearby points on a manifold. Roughly speaking, the former gives the distances of the spacetime while the latter defines directions of the spacetime. The formulation of GR divides into two cases depending whether the connection is handled independently of the metric tensor [34]. In the so called metric formulation the metric tensor alone determines the connection. Requiring the connection to be torsion free,

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{(\mu\nu)}, \tag{2.1}$$

and metric compatible,

$$\nabla_{\rho} g_{\mu\nu} = 0, \tag{2.2}$$

results in a connection related to the metric as

$$\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_{\mu} g_{\nu\rho} + \partial_{\nu} g_{\rho\mu} - \partial_{\rho} g_{\mu\nu} \right). \tag{2.3}$$

This is called the Christoffel connection. In contrast to the metric formulation, the Palatini formulation of GR assumes the connection to be independent of the metric tensor [34]. In that case the relation between the metric and the connection depends on the equations of motions and precise form of the theory. If the matter content of the universe does not couple to the connection nor the derivatives of the metric, and the connection is symmetric, the two formulations of the GR are physically equivalent [35].

The curvature of spacetime is given by the curvature scalar $\mathcal{R}$, also known as the Ricci scalar. The curvature scalar is formed from the metric and the Riemann tensor $\mathcal{R}^{\rho}_{\sigma\mu\nu}$ which is determined from the connection. The curvature scalar is defined as

$$R = g^{\mu\nu} R^{\lambda}_{\mu\lambda\nu}, \tag{2.4}$$

$$R^{\rho}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}. \tag{2.5}$$

Given by the metric, the connection and the curvature scalar, the equations of motion of the simplest form of general relativity follows from the Einstein Hilbert action

$$S_H = \int d^n x \sqrt{-g} \left(\frac{M_P^2}{2} R + \mathcal{L}_{\mathrm{mat}}(\Phi_i, \partial_\mu \Phi_i) \right). \tag{2.6}$$

Here g is the determinant of the metric tensor and $\mathcal{L}_{\mathrm{mat}}(\Phi_i, \partial_\mu \Phi_i)$ contains the matter part of the action. According to the Principle of Least Action the variation of the action

$$\frac{\delta S_H}{\delta X} = 0, \qquad X = g_{\mu\nu}, \Gamma, \Phi_i, ... \tag{2.7}$$

with respect to the degree of freedom X gives the equation of motion for the corresponding variable. Especially, variation with respect to the metric gives the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{2.8}$$

where $T_{\mu\nu}$ is the energy-momentum tensor built up from the matter content of the universe,

$$T_{\mu\nu} = -2 \frac{1}{\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta g^{\mu\nu}}. \tag{2.9}$$

In addition to the metric equation (2.8) there is a connection equation in the Palatini formalism that relates the connection to the metric [34]. The Palatini formulation is simpler in a sense that all dynamics follows from the variation with respect to the independent degrees of freedom. In contrast, in the metric formalism, having a well defined variation of action with respect to the metric requires adding an extra boundary term to the action [4]. This boundary term, called the York-Gibbson boundary term, ensures that the boundary of the manifold $\partial\mathcal{M}$ stays fixed in variation. Nevertheless, in the case of the Einstein-Hilbert action, the metric and the Palatini formulation are physically equivalent. That is, the solution to the connection equation is the Christoffel connection while the equation of motions for the metric and for the other fields equal in both formalisms.

Generalized action

The deviations from the Einstein-Hilbert cosmology are motivated by both cosmological observations and particle physics (see e.g. [34]). As discussed earlier, the accelerating expansion of the late universe and primordial inflation are not explained within the pure Einstein-Hilbert cosmology. Thus they might require modified gravitation. What is more, the particle physics approach to gravity is problematic as the proper quantum field description of gravity is missing. This manifests as nonrenormalisability or non-perturbativity when quantum corrections of the curvature scalar are taken into account. Indeed, the first order quantum corrections to the action (2.6) require addition of higher order terms in the Riemann tensor [36]. From these, the only stable terms appear to depend only on some powers of the curvature scalar. Therefore, expanding the action to be a general function of the curvature scalar $f(\mathcal{R})$ produces a simple, theoretically and observationally motivated model for the modified general relativity [34].

Let us then consider the action where the gravitational part is more general. On top of the arbitrary dependence on the curvature scalar

$$f(\mathcal{R}) = ... + \frac{\alpha_2}{\mathcal{R}^2} + \frac{\alpha_1}{\mathcal{R}} + \alpha_0 + \mathcal{R} + \beta_2\mathcal{R}^2 + \beta_3\mathcal{R}^3 + ... \tag{2.10}$$

let the curvature scalar be coupled to the some scalar field. The latter generalisation is an example of the scalar-tensor theories and has its implementation in Higgs inflation, discussed in detail in the following chapters.

The action for the combination of $f(\mathcal{R})$-theory of gravity and scalar-tensor theory reads

$$S = \int d^n x\sqrt{-g}\left(\frac{1}{2}f(R) + \frac{1}{2}q(h)R + \mathcal{L}_{\rm mat}\right). \tag{2.11}$$

In the case $f(\mathcal{R}) = \mathcal{R}$, $q(h) = \xi h^2$ the action is equivalent to Higgs inflation, and in the case $f(\mathcal{R}) = \mathcal{R} + \alpha\mathcal{R}^2$, $q(h) = 0$ it is equivalent to Starobinsky inflation. These two inflationary models will be considered more in detail in the following chapters. In addition, the choice of gravitational degrees of freedom give rise to different physical descriptions for the action (2.11) as the metric and the Palatini formulation of GR differ in this case [35].

In the metric formulation of GR the Einstein equation reads

$$\mathcal{R}_{\mu\nu}(f' + q) - \frac{1}{2}g_{\mu\nu}\left(f + q\mathcal{R}\right) - \left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box\right)(f' + q) = T_{\mu\nu}, \tag{2.12}$$

where f is derivated with respect to the Ricci scalar and the covariant derivative ∇ depend on the connection as [4]

$$\nabla_\mu\omega_\nu = \partial_\mu\omega_\nu + \Gamma^\lambda_{\mu\nu}\omega_\lambda. \tag{2.13}$$

In the Palatini case the metric and the connection equations are

$$(f' + q)\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(f + q\mathcal{R}) = T_{\mu\nu}, \tag{2.14}$$

$$\tilde{\nabla}_\kappa(f' + q)\sqrt{-g}g_{\mu\nu} = 0. \tag{2.15}$$

Here the covariant derivative $\hat{\nabla}$ in the Palatini case differs from the corresponding derivative in the metric case due to different connections. The solution to the connection equation in the Palatini case fixes the the relation between the metric and the connection to a form

$$\Gamma = \tilde{\Gamma} g_{\mu\nu}(f' + q), \tag{2.16}$$

where $\tilde{\Gamma}$ is the Christoffel connection (2.3).

Conformal transformation

In order to study physical predictions of the theory, the gravitational part of the action considered should be transformed to Einstein-Hilbert gravity (2.6). This is called the Einstein frame. The action where the generalized curvature terms are introduced, is called the Jordan frame action. The conventional procedure is to move from the Jordan frame to the Einstein frame action by suitable field redefinition. This is done by conformal transformation that generally transfer from one frame to another. It is a local change of scale that preserves the causal structure of a manifold but mixes up the gravitational and matter degrees of freedom. Hence, by moving from the Jordan frame to the Einstein frame, the matter sector of the actions completely changes as the gravitational part simplifies. The required change in the metric (2.11) is

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}, \qquad \Omega^2(x) = f' + q. \tag{2.17}$$

In general the conformal transformation provides the transformation function Ω to be any smooth non-vanishing function of the spacetime coordinates.

Moving to the Einstein frame the parameters of action change as

$$\sqrt{-\tilde{g}} = \Omega^4 \sqrt{-g} \qquad \tilde{\mathcal{R}} = \begin{cases} \frac{1}{\Omega^2}\left(\mathcal{R} + \frac{6}{\Omega}\Box\Omega\right) & \text{metric} \\ \Omega\mathcal{R} & \text{Palatini} \end{cases}. \tag{2.18}$$

Here the simple relation between the Jordan and Einstein frame curvature scalars in the Palatini formalism follows from the independence between the Riemann tensor and the metric. With these redefinitions the Einstein frame action is attained, but with the modified potential and kinetic terms.

Before writing the resulting Einstein frame action, let us consider the scalar degree of freedom that emerges from the non-trivial spacetime curvature term $f(\mathcal{R})$. One motivation for such a new scalar comes from the problematic higher derivatives in the Einstein equation (2.12). The appearance of the new scalar is seen by writing the dynamically equivalent action as (2.11),

$$S = \int d^n x \sqrt{-g} \left(\frac{1}{2} \left[f(\chi) + f'(\chi)\left(\mathcal{R} - \chi\right) + q\mathcal{R} \right] + \mathcal{L}_{\text{mat}} \right), \tag{2.19}$$

and varying it respect to the field χ. By doing so one obtains [34]

$$f''(\mathcal{R})(\mathcal{R} - \chi) = 0 \tag{2.20}$$

leading to $\chi = \mathcal{R}$ and the original action (2.11). By further denoting $\phi = f'(\chi)$ and adding the potential

$$W(\phi) = \chi(\phi)\phi - f(\chi(\phi)), \tag{2.21}$$

in the matter section $\mathcal{L}_{\rm mat}$, the Jordan frame action can be presented in the form

$$S = \int d^n x\sqrt{-g}\left(\frac{1}{2}\left(\phi + q\right)\mathcal{R} + \mathcal{L}_{\rm mat}\right). \tag{2.22}$$

The action (2.22) actually coincides with the Brans-Dicke action, one of the earliest scalar-tensor theory models [37]. In terms of the scalar ϕ the Einstein frame action reads

$$S_{\rm E} = \int d^n x\sqrt{-g}\left[\frac{1}{2}\tilde{\mathcal{R}} - \frac{3\gamma}{4\Omega^4}\left(\tilde{\partial}_\mu(\phi+q)\right)\left(\tilde{\partial}^\mu(\phi+q)\right) + W(\phi,q) + \frac{1}{\Omega^4}\mathcal{L}_{\rm mat}\right], \tag{2.23}$$

with the potential that contains the original part $V(\phi)$ and the part (2.21) arising from the conformal transformation, that is

$$\hat{V}(\phi, q) = \frac{V(\phi) + W(\phi)}{\Omega^4}. \tag{2.24}$$

Here $\gamma = 1$ in the metric case while $\gamma = 0$ in the Palatini case. In the Palatini formalism the field ϕ is not a dynamical degree of freedom due to the lack of kinetic terms. This means that, for example, an $\mathcal{R}^2$ based scalar-field inflation is not possible in the Palatini formulation of GR, in contrast to the metric formalism. In the metric formalism, the kinetic terms of the scalar field remain but of the non-canonical form. This means that the coefficient of the derivatives $\partial_\mu\phi\partial^\mu\phi$ is field dependent and deviates from the conventional constant value. To have canonical kinetic terms in the metric case requires one to once again make suitable field redefinition.

FRW metric

Finally, let us consider the parametrisation of the metric degrees of freedom in the background universe. The degrees of freedom for the metric that describe the spatially homogeneous and isotropic universe are the time-dependent scale factor $a(t)$ and spatial curvature parameter K. The latter can have the values $K = -1, 0, 1$ corresponding to an open, flat or closed universe. The relative change in the scale factor is called the Hubble rate

$$H \equiv \dot{a}/a, \tag{2.25}$$

and it characterizes the rate of expansion of the universe. The comoving size of the universe, *i.e.* the physical size of it scaled to the todays value a_0, is roughly given in terms of the Hubble parameter as $l^c \sim aH^{-1}$. At observed value of the Hubble parameter at the present epoch is $67.8 \pm 0.9\frac{\rm km/s}{\rm Mpc}$ [38]. In terms of the scale factor and the spatial curvature parameter the metric can be written as

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right). \tag{2.26}$$

This is called the Friedmann-Robertson-Walker (FRW) metric. Then, given the energy content of the universe, the Einstein equation fixes the time evolution of the scale factor. In the perfect fluid approximation the energy-momentum tensor has the diagonal form

$$T^{\mu}_{\nu} = \mathrm{diag}(-\rho, p, p, p), \tag{2.27}$$

where ρ is the energy density and p is the pressure of the fluid. For this, the resulting equations, Friedmann equations, are

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{2.28}$$

2.2 Standard Model Higgs

Consider the matter part of the action, $\mathcal{L}_{\mathrm{mat}}$, which is obtained from the flat spacetime field theory using the equivalence principle. This, the Standard Model of particle physics (SM), is a quantum field theory constructed from the classical field theory by the canonical quantization procedure. The particles of the theory are identified with the quantum excitations of the quantized fields.

The symmetry group of the quantum field theory completely determines it, the form $\mathcal{L}_{\mathrm{SM}}$ being the most general one obeying the given symmetries. The gauge symmetry group of SM is $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$. Here $SU_C(3)$ is the colour symmetry corresponding to strong interactions that acts only between quarks and gauge bosons of $SU_C(3)$. Further, the factor $SU_L(2) \otimes U_Y(1)$ describes the electroweak interactions between quarks, leptons and gauge bosons γ, W and $Z^{\pm}$. Among the particles of the Standard Model there is also the Higgs field, required for symmetry breaking.

Higgs mechanism

The Lagrangian that obeys the gauge symmetry $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ contains only massless gauge fields in symmetric phase. But if the gauge fields W, $Z^{\pm}$ mediating the weak force are massless, the interaction displays a long range behaviour with roughly the strenght as the electromagnetic force. In short, the presence of the Higgs field solves the problem by generating a ground state of the system that no more obeys the symmetries of the original Lagrangian: the symmetry group changes as $SU_L(2) \otimes U_Y(1) \to U_{EM}(1)$. The mechanism is called spontaneous symmetry breaking and as a consequence the massless gauge bosons absorb the degrees of freedom of the Goldstone-bosons of the theory and acquire masses [9–11].

To review the Higgs mechanism consider first the Higgs doublet that can be written as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho \end{pmatrix} e^{i\alpha_j \sigma^j/2}, \tag{2.29}$$

where α_j is a scalar field and σ_j Pauli's spin matrices [5]. When renormalisability and the $SU(2) \otimes U(1)$ invariance are taken into account, the most general Lagrangian for the Higgs field is

$$\mathcal{L}_{\text{Higgs}} = -(D_\mu H)^\dagger D_\mu H - \kappa^2 H^\dagger H - \lambda \Big(H^\dagger H \Big)^2. \tag{2.30}$$

Here the covariant derivative depends on the gauge bosons W_μ and B_μ and the generator Y of the hypercharge group $U_Y(1)$ as

$$D_\mu = \partial_\mu - \frac{i}{2} g W_\mu^j \sigma_j - i g' B_\mu Y. \tag{2.31}$$

If $\kappa < 0$ in the Lagrangian (2.30) the system has a ground state where the Higgs is different from zero,

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix}, \qquad \nu = \sqrt{\frac{-\kappa^2}{\lambda}} \tag{2.32}$$

with VEV $\nu \simeq 246$ GeV. Now the Higgs field can be rewritten around the new ground state in a unitary gauge as

$$H = \langle H \rangle + \delta H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + h \end{pmatrix}, \tag{2.33}$$

where h is the excitation of the Higgs field, the Higgs boson. The potential for the scalar field h becomes

$$V(h) = -\kappa^2 h^2 + \frac{1}{4} \lambda h^4. \tag{2.34}$$

Now the effective mass corresponding to the potential (2.34) in the ground state is $m_H^2 = \sqrt{-2\kappa^2}$, which has the experimental value $m_H = 125.09 \pm 0.21 \pm 0.11$ GeV [39]. Furthermore, the VEV related part of the kinetic term in (2.30) results in the masses

$$m_W = \frac{g\nu}{2}, \qquad m_{Z^\pm} = \frac{\nu}{2} \sqrt{g^2 + g'^2} \tag{2.35}$$

for the gauge bosons. The spontaneous symmetry breaking described above took place at the electroweak transition in the early universe when the temperature fell below ~ 100 GeV (see e.g. [40–42]).

Radiative corrections

Let us introduce the most important concepts of Quantum Field Theory (QFT) and Thermal Field Theory for Higgs inflation: the perturbative treatment of quantum corrections and the resulting running of couplings, and the thermal corrections to the Higgs effective potential.

The Standard Model of particle physics cannot be the complete theory. There is a number of observations that are not explained within the model, such as the existence of dark matter, neutrino masses, baryon and lepton asymmetries or inflation. Therefore, the SM is an effective field theory, meaning that at energies above the cutoff scale $\Lambda_{\rm UV}$, which could be the Planck mass M_P, or at distances smaller than $\Lambda_{\rm UV}^{-1}$, the theory does not yield a good description of Nature. The observations mentioned could be explained by a UV complete extended SM or by a more fundamental theory, such as string theory.

Below the cutoff scale $\Lambda_{\rm UV}$, the Standard Model is renormalisable and perturbative. The QFT running introduces energy scale dependence for the couplings of the theory. This is obligatory in order to avoid divergences in physical quantities. To see where these divergences arise, recall that in perturbation theory $\phi \to \phi_{\rm cl} + \delta\phi$ where $\phi_{\rm cl}$ is the classical background field and $\delta\phi$ is the quantum correction. With expanding the field equation to first order (one-loop) with respect to the small quantum correction results in the change [5]

$$V'(\phi) = V'(\phi_{\rm cl}) + V'''(\phi_{\rm cl})\langle\delta\phi^2\rangle. \tag{2.36}$$

Here V is the potential and the expectation of quantum correction can be written as [5]

$$\langle\delta\phi^2\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3\omega_k(\phi)}\left(\frac{1}{2} + n_k\right), \tag{2.37}$$

where n_k corresponds to the number of particles with momentum k. The integral (2.37) clearly diverges, already in a vacuum $n_k = 0$. For this, the potential acquires a divergent correction of the form

$$V(\phi) = V(\phi_{\rm cl}) + \frac{1}{(2\pi)^3}\int d^3\mathbf{k}\sqrt{k^2 + m^2(\phi)}. \tag{2.38}$$

So roughly, the inclusion of arbitrarily large momenta k together with the non-zero vacuum energy density shows up as infinities in quantum field theory and originates from the relationship between quantum theory and its classical counterpart. But since there is nothing physical in this relation, one may remove the divergencies by adding suitable counterterms depending on some regulator. In the end, one may remove the regulator to end up with the one-loop effective potential [43]

$$V(\phi) = V(\phi_{\rm cl}) + \frac{m_{\rm eff}^4}{64\pi^2}\ln\left(\frac{m_{\rm eff}^2}{\mu^2}\right), \tag{2.39}$$

where $m_{\rm eff}^2$ is the square of the effective mass of the field. The last part in (2.39) is interpreted as a field-dependent vacuum energy shift due to quantum fluctuations of the field. This depends on the arbitrary energy scale μ at which all the physical parameters are defined. However, the physics should be independent of the choice of scale, meaning that the effective mass should adopt this scale dependence. For the Higgs potential (2.34) at large energies the effective mass is $m_{\rm eff}^2 \simeq \lambda h^2$ yielding

$$\beta_\lambda = \frac{\lambda^2}{\pi^2}, \qquad \beta_\lambda \equiv \frac{\partial\lambda}{\partial\ln\mu}. \tag{2.40}$$

Finally, let us add to the Lagrangian (2.30) Higgs coupling to the top quark (which as a heaviest quark yields largest contributions to the radiative corrections) and take into account the massive gauge bosons originating from the SSB [5]

$$\mathcal{L}_{\text{mat}} = \mathcal{L}_{\text{Higgs}} + m_W^2 W^\mu W_\mu + \frac{1}{2} m_Z^2 Z^\mu Z_\mu - \frac{y_t}{\sqrt{2}} \nu t \tilde{t}. \tag{2.41}$$

Now the scale dependence of the couplings is solved from the renormalization group equations, which in the one-loop level are given by [44]

$$\begin{aligned}
\beta_\lambda &= \frac{1}{4\pi^2}\left(24\lambda^2 - 6{y_t}^4 + \frac{3}{8}(2g^4 + (g^2 + {g'}^2)^2) + (-9g^2 - 3{g'}^2 + 12{y_t}^2)\lambda\right), \qquad (2.42)\\
\beta_{y_t} &= \frac{1}{4\pi^2}\left(\frac{9}{2}{y_t}^2 - \frac{9}{4}g^2 - \frac{17}{12}{g'}^2 - 8g_s^2\right),\\
\beta_{g'} &= \frac{41}{6(4\pi)^2}{g'}^3,\\
\beta_g &= -\frac{19}{6(4\pi)^2}g^3,\\
\beta_{g_s} &= -\frac{7}{(4\pi)^2}g_s^3.
\end{aligned}$$

The running of couplings gives rise to the Higgs self coupling λ that decreases monotonically to negative values at (instability) scale $\Lambda_\text{I} \simeq 10^9 ... 10^{11}$ GeV, the actual scale being sensitive to the SM parameters (see e.g. [45–48]). This is shown in figure 2.1 where the radiative corrections are taken into account up to three-loop precision with Higgs and top quark masses varied within their 2-σ bounds $m_h = 125.09 \pm 0.24$ GeV and $m_t = 173.21 \pm 1.22$ GeV [49].

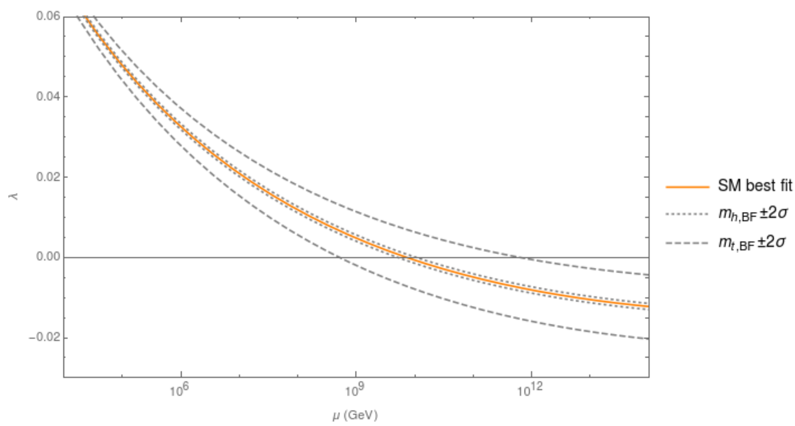

Figure 2.1: The running of Higgs self-interaction coupling constant with varied SM parameters.

For negative values of λ, the contribution to the energy density due to Higgs self-interaction becomes negative. If this term dominates, the total energy density becomes negative as well resulting in an unstable EW vacuum (see e.g. [16, 30, 44]. The situation is easily realised in the early universe where the Higgs field is supposed to acquire large values (see e.g. [15–17, 50–52]). Moreover, a ground state with negative energy can also be fatal for the present epoch as the Higgs field may tunnel over the potential barrier separating the electroweak and possible negative energy vacuums. However, as the longevity of the EW vacuum suggest, the tunneling probability should be very small with the tunneling rate exceeding the age of the present universe [16]. It is however possible that the exact values of SM parameters might lie away from their current central values by a few sigmas so that the instability scale is pushed close to the Planck scale [53]. There, at least, quantum gravity is supposed to modify the theory.

Temperature corrections

The form of quantum corrections in the effective field theory depend on the background state of the universe. The first order correction to the effective potential, computed in the previous section, was derived in the absence of background particles, $n_k = 0$. As is obvious from equation (2.37), the effective potential acquires also other contributions in the case $n_k \neq 0$. If the non-zero number density gives rise to a finite temperature, the Higgs effective potential acquires also termal corrections in addition to the quantum corrections introduced in the previous section. This might be a situation in the post-inflationary epoch when the inflationary field has decayed into the SM degrees of freedom producing the high temperature thermal plasma. The resulting thermal corrections may then remove the negative energy ground state of the Higgs potential [30].

The correction to the effective potential resulting for a thermal background of temperature T depends on the distribution of the particles n_k. Including both boson and fermion thermal equilibrium statistics, the one loop thermal correction to the Higgs effective potential can be wirtten as [54]

$$\Delta V(T,h) = T \sum_i \int \frac{d^3k}{(2\pi)^3} \ln(1 \pm e^{-\beta(k^2+(m_{\rm eff})_i^2)}) \ . \tag{2.43}$$

Here the plus sign corresponds to fermions and minus sign to bosons. For the Higgs field the dominant contributions come from the top quark and gauge bosons. For them, the thermal masses should be evaluated at the scale $\mu_t = 1.8T$ and $\mu_g = 7T$ respectively. Given the temperature of the thermal background, the correction (2.43) depends on the field value h through thermal masses. For the post-inflationary reheating stage, this temperature is approximated to be in the instant reheating [30]

$$T_{\rm reh} = \left(\frac{30 V_{\rm inf}}{g_* \pi^2}\right)^{1/4} , \tag{2.44}$$

where $g_* = 106.75$ is the effective number of degrees of freedom and $V_{\rm inf}$ is the value of the Higgs potential at the end of inflation. Here, the relation (2.44) follows from the approximation in which all the energy of inflation is instantly converted to the thermal plasma. In order to lift the potential from its global minimum to positive values, the condition

$$\Delta V(T_{\rm reh}, \chi_{\rm min}) > -V(\chi_{\rm min}) \tag{2.45}$$

should be fulfilled [1]. In practise, this constrains the inflationary energy to be high enough in order to provide high enough reheating temperature and consequently large enough corrections to the potential. Hence, the condition (2.45) limits the possible predictions of Higgs inflation as is discussed in the article I.

Chapter 3

Models of inflation with modified gravity

Inflation is driven by a general mechanism, which in the very early universe generates the initial conditions necessary for the observed universe. Homogeneity and isotropy, spatial flatness, absence of cosmological relics, and a hot thermal plasma with small inhomogeneities therein would appear to be extremely exceptional conditions for spacetime without the powerful mechanism of inflation.

The precise dynamics of inflation is not known. In the simplest case it follows from the energy dominance of some scalar field. After introducing the basic mechanism of inflation and the related observables, this chapter considers two inflationary models with minimal extensions of GR that agree well with the observations: Starobinsky inflation and Higgs inflation. Both models are motivated by quantum corrections in curved spacetime, introducing one new degree of freedom.

3.1 Motivation, mechanism and observables

Cosmological inflation refers to the epoch of accelerated expansion of the early universe

$$\text{inflation} \quad \Leftrightarrow \quad \ddot{a} > 0 \quad \Leftrightarrow \quad \frac{d}{dt}(aH)^{-1} < 0. \tag{3.1}$$

Here, the distance scale $(aH)^{-1}$ approximately gives the comoving size of the universe at the time t, the horizon. During the whole history of the universe, beginning from times of the early Big Bang epoch, the size of the comoving horizon has been growing. Indeed, the size of the observable universe has been enormously smaller at the time of decoupling than at the present time with

$$\frac{(a_{\text{dec}}H_{\text{dec}})^{-1}}{(a_0 H_0)^{-1}} \ll 1. \tag{3.2}$$

Still, as is revealed by CMB observations, the universe remains homogeneous over the whole observable region of $a_0 H_0$ at the time of decoupling. Hence, to explain this homogeneity over the causally disconnected regions, the inflationary period (3.1) offers a generic solution.

It is mainly due to this shrinking of the observable universe, that also the spatial flatness becomes explained. In terms of the density parameter

$$\Omega = \frac{\rho}{3M_P^2 H^2}, \tag{3.3}$$

the Friedmann equation (2.28) becomes

$$|\Omega - 1| = \frac{K}{aH}. \tag{3.4}$$

This is the equation for the time dependence of spatial curvature : the value $\Omega = 1$ corresponds to the case of a precisely flat universe. As the expansion of the universe has been decelerating over the last 10^{10} years, even the small initial deviation from the flat case would have grown to a sizeable value. In other words, the observed value of today, $\Omega_0 \sim \mathcal{O}(1)$, requires at the time of Big Bang Nucleosynthesis

$$|\Omega_{\rm in} - 1| < 10^{-16}, \tag{3.5}$$

which is, once again, attained easily with inflation (3.1).

In addition to the initial flatness and homogeneity condition, inflation explains the absence of the unwanted cosmic relics. Such relics, like magnetic monopoles or cosmic strings, may be easily produced in the spontaneously broken Grand Unified Theory phase transition occurring at temperatures $T \sim 10^{14}$ GeV. Driving the number density of these relics to practically zero, inflation is able to explain the lack of these relics in the observable universe.

The most wondrous effect of inflation is however the generation of small density perturbations in the universe. These perturbations have acted as seeds for all structure in the late universe. The capability of inflation to produce density inhomogeneties is due to nearly exponential expansion that stretches the small quantum fluctuations to the scales of horizon making them classical density perturbations. The review of the quantitative description of inflationary perturbations requires the definition of the inflationary mechanism and the use of cosmological perturbation theory, and is hence postponed to later subsections.

All the successful consequences of inflation, as described above, also set constraints for different models for inflation. The observations on the CMB fix the amplitude of scalar perturbations generated by inflation as well as the minimum amount of inflationary expansion. The various observables and their predicted values are reviewed in the final part of the section.

General mechanism of inflation

The general condition for inflation (3.1) requires the total pressure of the universe to be negative, as is seen from the Friedmann equation (2.28). In the case of an ideal cosmic fluid that feels only gravitation, the pressure always stays positive. However, for a scalar field with attractive interactions, like $\lambda\phi^4$, the pressure is negative. The energy domination of such a scalar field thus gives rise to the inflationary phase of the universe.

Let us consider the dynamics of an energetically dominant and homogeneous scalar field ϕ. The Lagrangian for such a field in the action (2.6) reads

$$\mathcal{L} = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + V(\phi), \tag{3.6}$$

and the energy momentum tensor is given by

$$T^{\mu\nu} = -\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\partial^\mu\phi + g^{\mu\nu}\mathcal{L}. \tag{3.7}$$

Further, the corresponding energy density and the pressure for the homogeneous field ϕ (i.e. $\nabla^\mu\phi = 0$) are

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3.8}$$

$$p = \frac{1}{2}\dot{\phi} - V(\phi). \tag{3.9}$$

With the use of equations (3.6), (3.7) and (3.8), the variation of the action (2.6) with respect to the field ϕ gives the equation of motion for the homogeneous scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{3.10}$$

The second term here results from the curvature of spacetime and acts like a friction slowing down the evolution of the field ϕ. This "slow roll" of the field guarantees efficient enough inflation that leads to the observable universe. Indeed, by approximating the energy density of the field to remain nearly constant and taking the friction term to dominate over the acceleration term in (3.10), the Friedmann equation (2.28) and the field equation (3.10) simplify to

$$H^2 = \frac{V(\phi)}{3M_P^2}, \tag{3.11}$$

$$3H\dot{\phi} = -V'(\phi). \tag{3.12}$$

These are referred as the slow roll equations. Furthermore, the regime of the inflationary potential that is flat enough to maintain (3.11) is determined by the smallness of the slow-roll parameters

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 < 1, \tag{3.13}$$

$$\eta \equiv M_P^2\frac{V''}{V} < 1. \tag{3.14}$$

In the regime where the slow-roll parameters are exactly zero, the Hubble parameter is constant, yielding exponential expansion, $a = e^{H_* t}$. This is called de Sitter space. To offer some exit from the inflationary state, it is assumed that, the expansion of the universe is only nearly exponential. If so, the actual amount of inflation can be discussed in terms of the number of e-folds

$$N \equiv \ln\frac{a_{\rm end}}{a_{\rm in}} \underset{\rm slowroll}{\approx} \frac{1}{M_P^2}\int_{\phi_{\rm end}}^{\phi_{\rm in}}\frac{V}{V'}d\phi, \tag{3.15}$$

where the subscripts refer to the end and beginning of inflation. In order to explain the homogeneity of the observable universe, the scales $k_0 = a_0 H_0$ of the present horizon must have been in causal contact during inflation. This is accomplished for

$$N > 61 - \Delta N_{\rm reh} + \frac{1}{4}\frac{U_*}{U_{\rm end}}. \tag{3.16}$$

Here, the term $\Delta N_{\rm reh}$ refer to the number of e-folds produced during the post-inflationary reheating state of the universe and U_* is the value of the potential at the pivot scale $k_* = 0.05~\mathrm{Mpc}^{-1}$.

Perturbations

The structure in the universe is assumed to originate from the small inhomogeneities produced by inflation. Following the comoving scale k, the evolution of perturbations can be divided into the initial state of quantum fluctuations at $k \gg aH$, their growth to the size of horizon $k = aH$, and further their freezing on superhorizon scales $k \ll aH$. After inflation, when radiation or matter dominate, the analysis of structure formation turns into the description of perturbations in different energy components. However, for the diagnostic of inflation models it is enough to consider the relation between the superhorizon perturbations to the primordial quantum field fluctuations. This allows the observational quantities to be expressed in terms of the parameters characterising the inflationary potential.

Let us first consider briefly the cosmological perturbation theory in a single-field case in a spatially flat universe. Due to the statistical homogeneity and isotropy of the universe, the perturbed quantity can be divided into a mean value whose evolution follows the exact FRW solution, and a small perturbation that averages to zero over the space. For example, for the inflationary field we have

$$\phi(\mathbf{x}, t) \to \bar{\phi}(t) + \delta\phi(\mathbf{x}, t), \tag{3.17}$$

with

$$\langle \delta\phi \rangle = 0 \qquad \text{and} \qquad \langle \delta\phi^*_{\mathbf{k}} \delta\phi_{\mathbf{k'}} \rangle = \frac{1}{V}\delta_{\mathbf{kk'}} P_{\delta\phi}(k), \tag{3.18}$$

reflecting the statistical FRW properties. Here the second equation gives the two-point correlation function of the perturbation (in Fourier space) in terms of the power spectrum

$$\mathcal{P}(k) \equiv \frac{k^3}{2\pi^2} V \langle |\delta\phi|^2 \rangle. \tag{3.19}$$

In addition to the field perturbations, perturbations in the metric tensor should also be considered. In general, the metric perturbations are decomposed as

$$ds^2 = -(1 + 2A)dt^2 + 2aB_i dt dx^i + a^2 \left[(1 - 2\phi)\delta_{ij} + 2E_{ij}\right] dx^i dx^j, \tag{3.20}$$

where the functions A, B_i, ψ and E_{ij} together include ten degrees of freedom. Four of these correspond to scalar, four to vector, and two to tensor degrees of freedom. Moreover, the choice of coordinate system fixes four of the ten degrees of freedom. The different choices are called different gauges. To give an example, the comoving gauge is the one that is defined so that the constant space-time coordinates follow the fluid flow lines, and the constant time slice hypersurfaces are orthogonal to them.

For one gauge choice, the relation between the mean value and the perturbation is fixed. Thus, different gauge choices correspond to different values of perturbations. A useful quantity, that stays constant both in gauge transformations and on superhorizon scales[1], is the comoving curvature perturbation. It describes how curved the constant time slices are in the comoving gauge. In terms of the inflationary perturbations, it is given by

$$\mathcal{R} = -H\frac{\delta\phi}{\dot{\phi}}, \qquad \mathcal{P}_{\mathcal{R}} = \left(\frac{H}{\dot{\phi}}\right)^2 \mathcal{P}_{\delta\phi} \tag{3.21}$$

The evolution of the different types of metric perturbations is as follows. The scalar perturbations are described by the power spectrum of the comoving curvature perturbations. In turn, the vector perturbations are found to decay as a^{-1} after horizon exit so — even if large initially — they become quickly negligible. The tensor perturbations, on the other hand, turn out to be gauge invariant and stay constant outside of the horizon. They are physically interpreted as gravitational waves. The primordial spectrum is defined similarly to (3.19), and depends on the amplitude of the gravitational wave h as

$$\mathcal{P}_h(k) \equiv 4\frac{V}{2\pi^2}k^3\langle|\delta h_{\mathbf{k}}|^2\rangle. \tag{3.22}$$

Let us finally introduce the cosmological observables. The relation of the two power spectra is called the tensor-to-scalar ratio

$$r \equiv \frac{\mathcal{P}_h}{\mathcal{P}_{\mathcal{R}}}, \tag{3.23}$$

while the spectral indexes of primordial scalar and gravitational waves read

$$n_s - 1 \equiv \frac{d\mathcal{P}_{\mathcal{R}}}{d\ln k}, \tag{3.24}$$

$$n_t \equiv \frac{d\mathcal{P}_h}{d\ln k}. \tag{3.25}$$

It turns out that for slow roll inflation primordial perturbations are nearly scale-invariant, meaning that the amplitude of the perturbations is nearly constant over the different length scales. Then, the spectral index remains close to a constant as well, and the scalar power spectrum can be written as

$$\mathcal{P}_{\mathcal{R}}(k) = A_s^2\left(\frac{k}{k_p}\right)^{n_s-1}, \tag{3.26}$$

where k_p is the pivot scale on which the observations are performed, corresponding to the time $a_pH_p = k_p$. To account for the small deviations from exact scale invariance, the running of the spectral index

$$\alpha_s \equiv \frac{dn_s}{d\ln k}, \tag{3.27}$$

[1] in the case of adiabatic perturbations

is usually included in the inflationary observables. The parameters A_s, n_s, r, and α_s together form the main observables of inflation. In order to relate them to the inflationary potential, the variance of inflationary perturbations in (3.19) needs to be solved.

Let us then consider the primordial inflationary perturbations. By substituting the perturbed inflationary field (3.17) into the field equation (3.10), the equation of motion for the perturbation becomes (in spatially flat gauge)

$$\delta\phi_{\mathbf{k}}'' + \frac{4}{a}\delta\phi_{\mathbf{k}}' + \left[\left(\frac{k}{aH}\right)^2 + \frac{m^2}{H^2}\right]\frac{\delta\phi_{\mathbf{k}}}{a^2} = 0. \tag{3.28}$$

This equation determines the evolution of perturbations up to the initial condition. Assuming that the initial state is of quantum origin, the field perturbation for $k \gg aH$ should be considered as the operator

$$\hat{\phi}(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\left[\phi_{\mathbf{k}}(t)\hat{a}_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{x}} + \phi_{\mathbf{k}}^*(t)\hat{a}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}\right]. \tag{3.29}$$

Then, deep inside the horizon, the initial value for the perturbation corresponds parametrically to the root mean square of the quantum field

$$\delta\phi_{\mathbf{k}}(t \to -\infty) = \sqrt{\langle 0||\hat{\phi}_{\mathbf{k}}|^2|0\rangle}. \tag{3.30}$$

With this, the overall solution for the inflationary field perturbation becomes

$$\delta\phi_{\mathbf{k}}(t) = V^{-\frac{1}{2}}\frac{H}{\sqrt{2k^3}}\left[\left(i + \frac{k}{aH}\right)\exp\left(\frac{ik}{aH}\right) + \left(i - \frac{k}{aH}\right)\exp\left(\frac{-ik}{aH}\right)\right]. \tag{3.31}$$

As the form of (3.31) reveals, the perturbations freeze to a constant after horizon exit $k \ll aH$, as already mentioned. Qualitatively, the quantum perturbations of the inflationary field are stretched to the large scales by the inflationary expansion making them classical perturbations. The primordial power spectrum for the comoving curvature perturbation on superhorizon scales becomes

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{\dot{\phi}}\right)\left(\frac{H}{2\pi}\right)^2. \tag{3.32}$$

Furthermore, during slow roll, this is directly related to the inflationary potential

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{24\pi^2}\frac{V/{M_P}^4}{\epsilon}, \tag{3.33}$$

as can be seen using equation (3.11).

Since the amplitude of gravitational waves acts like a scalar field during inflation, it acquires perturbations in a way similar to the inflaton field. In fact, the result (3.31) applies to the scalar field $(M_P^2/\sqrt{2})h$, hence giving the power spectrum

$$\mathcal{P}_h = \frac{8}{M_P^2}\left(\frac{H}{2\pi}\right)^2. \tag{3.34}$$

Finally, the power spectra (3.32) and (3.34), together with the use of the slow roll approximation, fixes the observables to depend on the inflationary potential as

$$\begin{aligned} n_s &= 1 + 2\eta - 6\epsilon && (3.35) \\ r &= 16\epsilon && (3.36) \\ \alpha_s &= 16\epsilon\eta - 24\epsilon^2 - 2\zeta. && (3.37) \end{aligned}$$

Here the running of the spectral index includes the third slow roll parameter

$$\zeta = M_P^4 \frac{V'}{V^2} V'''. \tag{3.38}$$

The values and limits for the observables at the pivot scale $k_p = 0.05\ \mathrm{Mpc}^{-1}$ measured by Planck are [38, 55, 56]

$$\begin{aligned} A_s &= 5.07 \cdot 10^{-5} && (3.39) \\ r &< 0.07 && (3.40) \\ n_s &= 0.9569 \pm 0.0077 && (3.41) \\ \alpha_s &= 0.011^{+0.014}_{-0.013}. && (3.42) \end{aligned}$$

Here, the value of tensor-to-scalar ratio in (3.39) is obtained by combining the Planck B-mode polarisation data with the Keck Array and BICEP2 data. The limits (3.39) will be used to constraint the inflationary models studied in the following chapters.

Multifield inflation

If during inflation there are several dynamically important scalar fields, adiabaticity and gaussianity of primordial perturbations may be lost.

Let us first consider adiabaticity. It means that the total energy density alone determines the total pressure, $\rho = \rho(p)$. Then, perturbations in different quantities can be expressed in terms of the total energy density perturbation. The perturbations violating this condition are called isocurvature perturbations or entropy perturbations. The total isocurvature perturbation is defined as

$$\mathcal{S} \equiv H \left(\frac{\delta p}{\dot{p}} - \frac{\delta \rho}{\dot{\rho}} \right), \tag{3.43}$$

and together with the comoving curvature perturbation it covers the evolution of scalar perturbations. The comoving curvature perturbation, which is proportional to the adiabatic field perturbation, is tangential to the field trajectory, and the isocurvature perturbation is perpendicular to it. As there is energy transfer between these modes, the comoving curvature perturbations do not necessarily stay constant outside the horizon. The measure of the relative energy transfer between the modes is the transfer angle

$$\cos\Theta = \frac{1}{\sqrt{1 + {T_{\mathcal{RS}}}^2}}, \tag{3.44}$$

which in the first approximation determines the change of the curvature power spectrum at super horizon scales as

$$P_{\mathcal{R}}(t) = P_{\mathcal{R}}(t_*) \cos^{-2}\Theta. \tag{3.45}$$

Further, the transfer angle is determined from the transfer function $T_{\mathcal{RS}}$ that together with the function $T_{\mathcal{SS}}$ gives the time evolution of scalar perturbations

$$\mathcal{R}(t) = \mathcal{R}(t_*) + T_{\mathcal{RS}}(t_*, t)\mathcal{S}(t_*), \tag{3.46}$$

$$\mathcal{S}(t) = \mathcal{R}(t_*) + T_{\mathcal{SS}}(t_*, t)\mathcal{S}(t_*). \tag{3.47}$$

The formal solution to these equations in terms of e-folds is

$$T_{\mathcal{RS}}(N_*, N) = -\int_{N_*}^{N} dN' A(N') T_{\mathcal{SS}}(N_*, N'), \tag{3.48}$$

$$T_{\mathcal{SS}}(N_*, N) = \exp\left[-\int_{N_*}^{N} dN' B(N')\right], \tag{3.49}$$

where $A(N)$, $B(N)$ are model dependent parameters, given for example in equations (3.36)-(3.40) in [57].

In addition to the transfer angle (3.44) the isocurvature fraction

$$\beta_{\rm iso} = \frac{T_{\mathcal{SS}}}{1 + T_{\mathcal{RS}} + T_{\mathcal{SS}}}. \tag{3.50}$$

may also be used to describe the effects of the isocurvature perturbation.

Another deviation from the single field case is the possible production of large non-gaussianity. In that case, the three-point correlator of primordial perturbation $\delta_{\mathbf{k}_i}$

$$\langle \delta_{\mathbf{k}_1} \delta_{\mathbf{k}_2} \delta_{\mathbf{k}_3} \rangle, \tag{3.51}$$

generally differs from zero and can be used to measure the level of non-gaussianity. One of the simplest types of non-gaussianity has the form

$$\Phi(\mathbf{x}) = \Phi_G(\mathbf{x}) - f_{NL}\Phi_G(\mathbf{x})^2, \tag{3.52}$$

where Φ is the Bardeen potential, i.e., the scalar metric perturbation in the conformal-Newtonian gauge, Φ_G is its gaussian value, and f_{NL} the local non-gaussianity parameter. The latter is a measure of the level of non-gaussianity. According to the Planck observations it is constrained to [58]

$$f_{NL}^{\rm local} = 0.8 \pm 5.0. \tag{3.53}$$

For the isocurvature perturbations, the Planck data sees no evidence. The bounds for the transfer angle Θ are given by [56]

$$-0.3 < \sin\Theta < 0.2. \tag{3.54}$$

3.2 Inflation with non-minimally coupled Higgs

The Higgs field serves as the most natural candidate for inflaton, as it is the only fundamental scalar field in the Standard Model. The potential of the Higgs field at inflationary energies is determined by the self-interaction

$$V(h) = \frac{\lambda}{4}h^4. \tag{3.55}$$

Unfortunately, the SM potential (3.55) alone does not lead to inflation compatible with the observed universe. The problem with the pure SM Higgs is the production of too strong inhomogeneities over the appropriate duration of inflation. To see this, consider the power spectrum (3.33) written in terms of the number of e-folds in (3.15),

$$\mathcal{P}_\mathcal{R} = \frac{2\lambda}{3\pi^2}(N+1)^3. \tag{3.56}$$

Now, the observed amplitude of scalar perturbations (3.39) together with the appropriate duration of inflation (3.16) requires the self-coupling to retain a values not in agreement with observations

$$\lambda \sim 10^{-13}, \tag{3.57}$$

ruling out inflation with pure SM Higgs. The situation remains also unchanged for the renormalisation group enhanced potential [59–61].

Fortunately, extending SM to curved spacetime, one may introduce a new term in to the action of the Higgs field [62, 63]. Due to the renormalisation of the energy-momentum tensor, the Higgs field is found to be non-minimally coupled to gravity and the Lagrangian is expanded to read

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{M^2}{2}R + \frac{1}{2}\xi h^2 R. \tag{3.58}$$

This is now a theory of extended gravity in the sense described in Chapter 2. The fact that the Lagrangian (3.58) may lead to an inflationary stage of the early universe was first suggested by Bezrukov and Shaposhnikov in 2007 [14]. Higgs inflation in this paper was considered in the metric formalism while the first study of it in Palatini formalism was performed in [64]. The authors of [14] found that if the value of the non-minimal coupling lies in the range $1 \ll \sqrt{\xi} \ll 10^{17}$, the model predicts both successful particle physics and inflation. Here, below the upper bound $\sqrt{\xi} \ll 10^{17}$, the low energy limit of the model reduces to the Standard Model, and the *a priori* arbitrary mass scale M can be identified with the Planck mass, $M = M_P$. On the other hand, above the lower bound $\sqrt{\xi} \gg 1$, the model is able to give rise for the suitable flat potential at high energies leading to successful inflation.

Let us summarize predictions for the observables in the classical approximation of Higgs inflation (3.58). The model has a high degree of predictability as it introduces only one new parameter to the SM, the non-minimal coupling ξ. Furthermore, since the Higgs field can be integrated out in the large-field regime and the couplings between the Higgs and other SM particles are known experimentally, the cosmological observables depend only on the duration of inflation.

Comparing the action (3.58) to the general action for extended gravity (2.11) one may identify (in units $M_P = 1$)

$$f(\mathcal{R}) = \mathcal{R} \tag{3.59}$$

$$q(h) = \xi h^2 \tag{3.60}$$

resulting in the conformal transformation

$$\Omega^2 = 1 + \xi h^2. \tag{3.61}$$

This gives the action

$$S_E = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \frac{\Omega^2 + p6\xi^2h^2}{2\Omega^4}\partial_\mu h\partial^\mu h - V(h)\right], \tag{3.62}$$

where $p = 0$ in the Palatini case $p = 1$ in the metric case. The potential reads [14]

$$V(h) = \frac{\lambda}{4}F(h)^4, \qquad F(h) \equiv \frac{h}{\sqrt{1+\xi h^2}}. \tag{3.63}$$

In the small-field regime $\xi h^2 \ll 1$ this reduces to the pure SM potential as the non-minimal coupling to gravity can be ignored. At the opposite regime $\xi h^2 \gg 1$, the potential approaches to a constant allowing for chaotic inflation type behaviour in the Higgs field [65].

In Higgs inflation the metric and the Palatini formulations result in different inflationary potentials as the kinetic terms in (3.62) differ. Transforming into canonical kinetic terms then applies a field reparametrisation different in these cases. The required reparametrisation is

$$\frac{d\chi}{dh} = \frac{\sqrt{1+\xi h^2 + p6\xi^2h^2}}{1+\xi h^2}, \tag{3.64}$$

together with the reparametrisation $\psi \to \Omega^{-3/2}\psi$ of the top quark. In the Palatini case the relation (3.64) can be solved exactly to yield the potential

$$F(\chi) = \frac{1}{\sqrt{\xi}}\tanh(\sqrt{\xi}h). \tag{3.65}$$

In the metric case the relation (3.64) obeys the asymptotic solutions [14]

$$F(\chi) = \begin{cases} \chi & , \ \chi \ll 1/\xi \\ \frac{1}{\sqrt{\xi}}\left(1 - \mathrm{e}^{-\sqrt{2/3}\,\chi}\right)^{1/2} & , \ \chi \gg 1/\sqrt{\xi} \end{cases}. \tag{3.66}$$

Even though the relation (3.64) determines the function $h(\chi)$ at the tree level also in the intermediate regime $1/\xi < \chi < 1/\sqrt{\xi}$ in the metric case, the resulting form is somewhat unreliable there [30]. This follows from the breakdown of the perturbative treatment of quantum corrections in this regime. A more detailed discussion of quantum corrections is given in the next chapter.

In the large field limit $\xi h^2 \gg 1$ the relation (3.64) between the fields h and χ simplifies to

$$\frac{d\chi}{dh} = \begin{cases} \frac{\sqrt{6}}{h} & \text{metric} \\ \frac{1}{\sqrt{\xi} h} & \text{Palatini} \end{cases}. \tag{3.67}$$

In the metric case this leads to slow roll observables

$$\epsilon \simeq \frac{4}{3}\frac{1}{\xi^2 h^4}, \qquad \eta \simeq -\frac{4}{3}\frac{1}{\xi h^2}, \qquad \zeta \simeq \frac{16}{9}\frac{1}{\xi^2 h^4}. \tag{3.68}$$

As follows from (3.67), in the Palatini case the first order slow roll parameters are multiplied by the factor 6ξ. Writing in terms of the number of efolds the observables in the metric case are

$$n_s = 1 - \frac{2}{N}, \qquad r = \frac{3}{4N^2}, \qquad \alpha_s = -\frac{2}{N^2}. \tag{3.69}$$

In terms of N in the Palatini formalism the spectral index n_s obeys the relation equal to the metric case (3.69) at the leading order while the tensor-to-scalar ratio is multiplied by the factor $1/6\xi$.

The observables (3.69) lie surprisingly well within the Planck one-sigma range (3.39) for the observable amount of efolds $N \approx 60$. In turn, the amplitude of scalar perturbations is not fixed from observations, but it rather fixes the ratio λ/ξ^2. Writing the power spectrum (3.33) in terms of N

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{24\pi^2}\frac{4}{3}\frac{\lambda}{\xi^2}N^2, \tag{3.70}$$

one obtains

$$\frac{\lambda}{\xi^2} \sim 10^{-8}. \tag{3.71}$$

This indicates the need for a large value of the non-minimal coupling — $\xi \simeq 2 \cdot 10^3$ for $\lambda = 0.1$ — for successful Higgs inflation. The large value of ξ also results in the tensor-to-scalar ratio being significantly smaller in the Palatini than in the metric case. This enables the exclusion of the Palatini Higgs inflation model if the tensor-to-scalar ratio large enough is detected in the future.

3.3 Inflation with R^2-term

The very first model for inflation was proposed by Strarobinsky in 1980 [27] (see also [66, 67]). The model expands the matter sector in the Einstein-Hilbert action (2.6) to include the one-loop quantum corrections of the free matter fields in the energy-momentum tensor, that is $T_{\mu\nu} \to \langle T_{\mu\nu} \rangle$. This modifies the action by giving rise to the squared curvature term. The gravitational part of the action becomes

$$f(\mathcal{R}) = \mathcal{R} + \alpha\mathcal{R}^2, \tag{3.72}$$

where α is a dimensionless constant. The non-singular self-consistent solution with this one-loop corrected energy-momentum tensor is the maximally symmetric de Sitter state with $a(t) \propto e^{H_0 t}$. The model can also be treated as a scalar field driven inflation as the $\mathcal{R}^2$ gives rise to the new degree of freedom, as was discussed in the first chapter. This holds only in the metric formalism. In the Palatini formalism the Eintein-Hilbert action plus $\mathcal{R}^2$term does not lead to an inflationary universe (see e.g. [68]).

For the Einstein frame solution (2.23) one finds the form

$$S_E = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \frac{3}{4\Omega^4}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right], \tag{3.73}$$

where the conformal transformation is

$$\Omega^2 = \phi, \tag{3.74}$$

and the potential is

$$V(\phi) = \frac{1}{8\alpha}\left(\frac{\phi-1}{\phi}\right)^2. \tag{3.75}$$

In order to have the kinetic term of (3.73) in the canonical form, a reparametrisation

$$\frac{d\phi}{d\psi} = \sqrt{\frac{2}{3}}\phi, \tag{3.76}$$

is required. Then, in terms of the field ψ, the potential acquires exactly the same form as in the case of the non-minimally coupled Higgs field. That is

$$V(\psi) = \frac{1}{16\alpha}\left(1 - e^{-\sqrt{\frac{2}{3}}\psi}\right)^2. \tag{3.77}$$

Of course, the equivalence of the potentials of Higgs and Starobinsky inflation applies only at the classical level. When the quantum correction of the inflatons are included the shape of the potential in the two cases differs. However, the tree-level predictions of Starobinsky inflation for the tensor-to-scalar ratio, spectral index and its running coincide to those of Higgs inflation, and are just given by (3.69). The observed value of amplitude of scalar perturbations restricts the value of α to

$$\alpha \sim 10^8. \tag{3.78}$$

Chapter 4

Higgs inflation with radiative corrections

The correct inclusion of quantum corrections in the inflationary model with a non-minimal coupling between the Higgs field and gravity would require a theory of quantum gravity. This shows up as the breaking of perturbative treatment of quantum corrections over the unitarity cutoff scale $1/\xi$. In the case of a large non-minimal coupling $\xi \gg 1$, suggested by observations (see (3.71)), the energy scale of inflation $1/\sqrt{\xi}$ fairly exceeds this cutoff scale. This would in the worst-case scenario lead to the loss of all predictability. However, in the case of the non-minimally coupled Higgs, the potential displays two distinct regimes where the model becomes *effectively* renormalisable, allowing reliable computations of quantum corrections over these regimes [44, 69–72].

The two effectively renormalisable regimes are the low energy SM regime and the high energy inflationary regime. In the regime close enough to the EW vacuum, the non-minimal coupling is simply negligible and can be ignored up to M_P/ξ [73, 74]. In the large field regime, in turn, the effective renormalisability is achieved due to the asymptotic flatness of the Higgs potential [70]. Thus, only at the intermediate energies between the two the model is subject to uncontrollable non-renormalisable effects [29, 30]. In Higgs inflation it is assumed that the loss of unitarity at the scale M_P/ξ is solved by the intermediate scale non-perturbative physics [70].

The overall impact of the non-renormalisable physics can be represented by effective jumps in the couplings of the model, i.e., unknown interpolations between the known couplings in the two renormalisable regimes [29–31]. These jumps cannot be determined by the perturbatively renormalisable physics. Furthermore, the inclusion of quantum corrections via these jumps may change the tree-level potential of the Higgs field in a way that strongly affects the cosmological predictions of the model. This happens if the potential contains a *feature* in the inflationary regime [1, 2, 29–31]. The main purpose of this chapter is to discuss the possible features and their cosmological signatures.

4.1 Sensitive model

Before discussing the effects of non-renormalisable physics on the observables, let us consider in this subsection the theoretical consistency conditions for Higgs inflation. The two problematic issues are the non-renormalisability and the naturalness of the model (see e.g. [1, 30, 31, 50, 75]). The former is related to the inclusion of quantum corrections, the latter to the validity of the quantum corrected model. These render Higgs inflation a sensitive albeit self-consistent model for which the computation of quantum corrections is a non-trivial task.

Breaking unitarity

Let us fist discuss the unitarity of Higgs inflation. The non-minimal coupling between the Higgs and gravity introduces the cutoff scale M_P/ξ above which the perturbative unitarity of the model is lost [73, 74]. This prevents perturbative computation of quantum correction over that scale. The quantum corrected Higgs potential obeys then one or the other of the two possible solutions: the Jordan frame Higgs Lagrangian (3.58) is either expanded by the non-negligible higher order operators

$$V(h) = \sum_{n\geq 4} \frac{1}{\Lambda^{2n-4}} \lambda_n(h) h^{2n}, \tag{4.1}$$

that push the perturbative unitarity cutoff over the inflationary energies, or, it obeys the non-perturbative solution that preserves unitarity [30].

In [73] it is shown that the flatness of the potential required by inflation is spoiled unless the coefficients of the higher order operators (4.1) in the Higgs Lagrangian are practically zero. The absence of a symmetry principle forbidding the presence of these operators makes Higgs inflation unnatural[1].

The absence of the higher order operators (4.1) also leaves the strong assumption of existence of the non-perturbative unitary solution for the quantum corrected Higgs potential [30, 70]. It makes Higgs inflation a strongly coupled model for which the correct inclusion of quantum corrections would demand non-perturbative physics in the regime $M_P/\xi < h < M_P/\sqrt{\xi}$. The motivation for this non-perturbative solution has been argued to follow from the self-cosistency of the model [30, 70].

Although unnatural, Higgs inflation seems to be self-consistent. Namely, the cut-off scale depends on the expectation value of the background field. When the Lagrangian is expanded around the EW vacuum, the unitarity breaks at $1/\xi$, but expansion around the large background field $\bar{h} \gg 1/\xi$, gives the unitarity bound such that the Hubble scale and reheating temperature are always below this scale [69, 76].

[1]Although, one such a symmetry could be the exact quantum scale invariance, but so far it is not known whether such a symmetry is realised in nature [30].

Inclusion of quantum corrections

Let us then discuss quantum corrections in the model of Higgs inflation. These corrections can be computed by perturbative quantum field theory only in the regimes where the non-minimal Lagrangian (3.58) correspond to the effective unitary theory, *i.e.* $h < M_P/\xi$ and $M_P/\sqrt{\xi} < h$. At these effectively renormalisable regimes, the generally non-polynomial form of the Higgs potential

$$V(\chi) = \frac{\lambda}{4} F[h(\chi)] \tag{4.2}$$

reduces to the polynomial form allowing for the perturbative treatment of quantum corrections.

The potential at the two effectively renormalisable regimes, the SM regime and the inflationary regime, is modified separately by the RGE running of the coupling λ. At the SM regime, λ runs according to (2.42) all the way to the unitarity cut-off scale M_P/ξ. On the other hand, at the inflationary energies $h > M_P/\sqrt{\xi}$, the running of λ follows the chiral RG equations. This is because the potential of the Higgs approaches to a constant, satisfying the approximate scale symmetry [77, 78]. As the potential becomes flat, the Higgs becomes massless and decouples from the other SM fields. The effective remaining model has no dynamical Higgs and resembles thus a chiral SM [70]. Even though the chiral SM is not renormalisable, it can be treated order by order and the leading correction is calculable. The chiral runnings of the couplings at one loop are [70]

$$\begin{aligned}
16\pi^2\beta_\lambda &= -6y_t^4 + \frac{3}{8}(2g^4 + (g'^2+g^2)^2) + \lambda(-3g'^2 - 6g^2 + 12y_t^2) \,, \\
16\pi^2\beta_{y_t} &= y_t\left(-\frac{17}{12}g'^2 - \frac{3}{2}g^2 - 8g_3^2 + 3y_t^2\right) \,, \\
16\pi^2\beta_{g_3} &= -7g_3^2 \,, \\
16\pi^2\beta_{g'} &= \frac{27}{4}g'^2 \,, \\
16\pi^2\beta_g &= -\frac{13}{4}g^2 \,.
\end{aligned} \tag{4.3}$$

As discussed in Chapter 1.2, in addition to the running of the couplings, the potential is modified by the logarithmic terms (see e.g. the one-loop correction in (2.38)). In a nonrenormalisable theory, the form of these terms depends on the renormalisation and subtraction schemes. Since the UV completion in Higgs inflation is not known, only assumptions on the UV completion can be made. As discussed in [77–79], the most important two assumptions guaranteeing inflation are the absence of heavy particles with masses larger than EW scale and the scale-invariance of the quantum corrected potential. The renormalisation scheme encompassing these assumptions is dimensional regularisation [48]. Together with $\bar{\mathrm{MS}}$ renormalisation scheme it gives the one-loop correction [70]

$$U_{1-\mathrm{loop}} = \frac{6m_W^4}{64\pi^2}\left(\ln\frac{m_W^2}{\mu^2} - \frac{5}{6}\right) + \frac{3m_Z^4}{64\pi^2}\left(\ln\frac{m_Z^2}{\mu^2} - \frac{5}{6}\right) - \frac{3m_t^4}{16\pi^2}\left(\ln\frac{m_t^2}{\mu^2} - \frac{3}{2}\right) \tag{4.4}$$

where the masses are

$$m_W^2 = \frac{g^2F^2}{4}, \qquad m_Z^2 = \frac{(g^2+g'^2)F^2}{4}, \qquad m_t^2 = \frac{y_t^2F^2}{2}. \tag{4.5}$$

The final thing to fix the form of the quantum corrections, is the choice of the renormalisation scale μ. In a renormalisable theory it can be chosen freely, but in a non-renormalisable theory it affects the size of the validity region of the loop correction in question. In [70, 78] two prescriptions were discussed. In prescription I, the renormalisation scale is field-independent in the Einstein frame, $\mu^2 \propto M_P^2$, leading to a field-dependent renormalisation scale in the Jordan frame. As opposed to this, in prescription II, the renormalisation scale is field-independent in the Jordan frame and field-dependent in the Einstein frame. In the research papers I and II we have chosen prescription I — in agreement with [78] — since it allows the asymptotic scale invariance to be valid also at the quantum level for large field values. In order to minimise the logarithmic contributions in (4.4), this choice gives

$$\mu^2 = \frac{\gamma^2}{2} m_t^2(\chi) = \frac{\gamma^2}{2} y_t^2 F^2(\chi), \tag{4.6}$$

where the constant γ takes into account contributions of other particles than the heaviest one, the top-quark. For numerical computations, we chose the value $\alpha = 1$ in article I whereas in the article II we solved it from the condition that the one-loop correction (4.4) vanishes at the hilltop.

4.2 The RGE improved inflationary potential

The fact that a perturbative computation of quantum corrections is not possible in the intermediate regime $1/\xi < h < 1/\sqrt{\xi}$ results in an ambiguity in the potential of the non-minimally coupled Higgs. As found in [29, 30] the non-renormalisable physics in this regime can be effectively parametrised in the jumps of the coupling constants — the couplings λ and y_t being the most relevant ones for the Higgs potential. Such an approach can also be theoretically motivated as it is connected to *a priori* unknown coefficients B_n in the dimensional regularisation [30]. These theoretical considerations suggest that the coupling λ jumps by $\Delta\lambda$ as

$$\lambda(\mu) = \lambda^{\rm SM}(\mu_0) + \Delta\lambda S(\mu(\chi)), \qquad S(\chi) \equiv \Big([F'(\chi)]^2 + F'''(\chi)F(\chi) \Big)^{-1} - 1 \tag{4.7}$$

in the intermediate region $1/\xi < h < 1/\sqrt{\xi}$. The step function $S(\chi)$ behaves as

$$S(\chi) \xrightarrow{h \ll 1/\xi} 0, \qquad S(\chi) \xrightarrow{h \gg 1/\sqrt{\xi}} -1. \tag{4.8}$$

After inclusion of the jumps in λ and y_t the inflationary potential depends now on the three unknown parameters $\Delta\lambda$, δy_t and ξ.

The jumps may have twofold consequences for the model. Firstly, they may stabilise the Higgs potential. If the Higgs self-interaction coupling λ runs to negative values, as discussed in Chapter 2, a large enough jump $\Delta\lambda$ may lift the Higgs field to positive values thus making inflation possible. The situation is illustrated in the figure 4.1. Secondly, the values of the jumps together with the SM parameters and ξ determine whether the tree level predictions of Higgs inflation are preserved. If the inflationary value $\lambda_{SM} + \Delta\lambda$ is small enough (and positive) the potential may include *a feature* such as an inflection point [1, 29–31, 50, 75, 80–82], a hilltop [31, 82] or a hillclimbing [83, 84] regime.

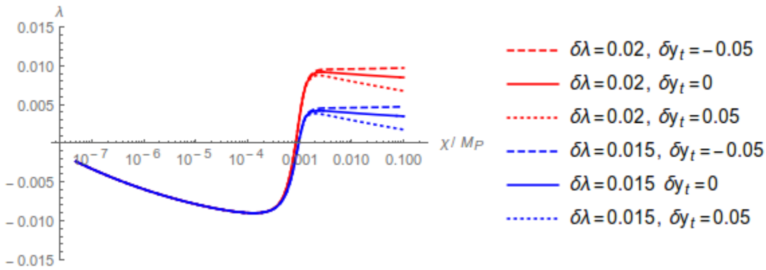

Figure 4.1: The running of λ affected by the jumps $\Delta\lambda$ and Δy_t.

Critical regime

After inclusion of the quantum corrections, the inflationary potential is

$$V(\chi) = \frac{\lambda(\mu)}{4} F^4(\chi) \ , \qquad \mu \propto F(\chi) \tag{4.9}$$

where λ is subject to the jump (4.7) and in the inflationary regime is determined from the chiral RG equations (4.3). In turn, in the SM regime the running of λ is mostly affected by the Higgs and top quark masses (m_h, m_t). Thus, the proper study of the inflationary predictions involves a variation over all the parameters $(m_h, m_t, \xi, \Delta\lambda, \Delta y_t))$.

The RGE running of λ makes the inflationary observables (3.35) λ-dependent. For most of the parameter space $(m_h, m_t, \xi, \Delta\lambda, \Delta y_t)$ the running of λ remains relatively small so that the tree-level predictions are preserved. However, for the fine-tuned values of these parameters the RGE running of λ, together with the jumps, may result in a change in λ that is of the order of λ. In such a case, one or both of the first two derivatives of the Higgs potential

$$V' = \frac{F^3 F'}{4} \left(4\lambda + \beta_\lambda\right) \tag{4.10}$$

$$V'' = \frac{(F^3F')'}{F^3F'} V' + \frac{F^2 F'^2}{4} \left(4\beta_\lambda + \frac{d\beta_\lambda}{d\ln\mu}\right) . \tag{4.11}$$

may accidentally vanish, resulting a feature in the potential.

There are basically two types of features. At the inflection point type feature the second derivative vanishes at some point at the inflationary regime, the coincidental vanishing of both derivatives $V'' = V' = 0$ being an extreme inflection point. This is considered in article I and earlier in [29–31, 50, 80, 81]. At the hilltop, only the first derivative vanishes resulting in a local maximum in the potential. This is considered in article II and earlier in [31, 82].

The study of the Higgs inflationary potential with a feature is relevant as it completes the search of all the possible predictions of the model. In addition, successful inflation requires the small slow roll parameters to agree with the observations (3.39). This is easily achieved in the vicinity of a feature as the derivatives (4.10), (4.11) are small. In the case of an inflection point both the slow roll parameters vanish at the feature whereas in the case of a hilltop only the slow roll parameter ϵ

vanishes at the hilltop. In the latter case, in order to achieve a successful end of slow roll inflation, the observable scales must have crossed the horizon at field values below the hilltop.

For the numerical investigations, the most practical way for scanning the parameter space is not to adopt the jumps $(\Delta\lambda, \Delta y_t)$ directly. Instead, one may use the parametrisation

$$4\lambda(\mu) + \beta_\lambda(\mu) \equiv \lambda(\mu)\delta_1(\mu), \tag{4.12}$$

$$4\beta_\lambda(\mu) + \frac{d\beta_\lambda(\mu)}{d\ln\mu} \equiv \beta(\mu)\delta_2(\mu) \ . \tag{4.13}$$

with which the smallness of the derivatives of the potential can be easily handled [1]. The magnitudes of the parameters δ_1 and δ_2 are related to the size of the feature — the inflection point $\delta_1 = \delta_2 = 0$ being an extreme. Confronted with such an exact condition one can fix one of the jumps to reduce the volume of the parameter space $(m_h, m_t, \xi, \Delta\lambda, \Delta y_t)$. For example, in article II, the condition $V' = 0$ was used to solve for the jump Δy_t. As the SM parameters were fixed as well, the parameter space consisted only of two free parameters. These were chosen to be the value of λ at the hilltop and the parameter δ_0 that is related to the field value at the hilltop by

$$\delta_0 = \frac{1}{\xi h_0^2}. \tag{4.14}$$

In both articles the jumps at the transition scale were solved by letting the couplings λ and y_t run from the inflationary scale and from the SM scale separately to the transition scale, and then comparing their values there.

4.3 Constraints

The domains in the parameter space $(m_h, m_t, \xi, \Delta\lambda, \Delta y_t)$ giving rise to successful inflation require both theoretical and observational constraints to be satisfied. These include the Planck ranges (3.39) of the observables to be maintained, the duration of inflation to be finite and the post-inflationary reheating to be such that relaxation of the Higgs to EW vacuum is possible.

The succesful end of inflation is achieved in the slow roll approximation if the potential monotonically decreases towards smaller field values. In a case of a hilltop this requires the observational 60 efolds to take place on the field values below the local maximum. In the case of an inflection point type feature only the cases, for which

$$V'(\chi) \geq 0 \ , \qquad \text{for} \quad \chi > \chi_{\text{end}}, \tag{4.15}$$

were accepted as they offer the graceful exit from the slow roll regime.

The strongest consistency constraint on the parameters $(m_h, m_t, \xi, \Delta\lambda, \Delta y_t)$ follows from the post-inflationary evolution of the Higgs field. As was discussed in section (1.2), the best-fit values of the Higgs and the top quark masses suggest that the coupling λ runs to negative values. In such a case, inflation can be realised only if the non-renormalisable physics at the intermediate scale results λ to jump on the positive values at the transition scale. However, this is not enough since the relaxation of the Higgs to EW vacuum requires high enough reheating temperature, *i.e.* high enough energy of the Higgs field at the end of inflation, see (2.44). The succesful reheating most strictly constrains the value of $\Delta\lambda$.

Finally, let us summarize the effects of various parameters determining the Higgs inflationary potential. The SM parameters, as discussed in the first chapter, control the scale where λ runs into negative values and thus affects the deepness of the global minimum. By increasing the value of the top mass, and decreasing the value of the Higgs mass, the negative energy minimum becomes deeper. Also, the value of ξ controls this minimum, as increasing it lowers the transition scale M_P/ξ where the jumps take place. On the other hand, ξ determines the energy scale of inflation with $V \propto \xi^{-2}$. Due to these two opposite effects, the condition of successful reheating is thus nontrivially dependent on ξ while the net result depends on the precise values of all the parameters. For the results of the systematic scan of the parameter space, see the Figure 1 in the paper I for a inflection point type features and the Figures 5 and 6 for a hilltop case.

Chapter 5

Higgs-Starobinsky inflation

Having considered two inflationary models — the Higgs and the Starobinsky model — both motivated by quantum corrections, it is natural to incorporate both in the same analysis. The resulting combined model involves then two scalar fields, the Higgs field and the gravitational degree of freedom of the Starobinsky model, the scalaron, that either alone or together are responsible for primordial inflation.

5.1 General picture

The dynamics of Higgs-Starobinsky inflation is fundamentally determined by the choice of the degrees of freedom. In the metric formulation of GR it is a two-field model of inflation while in the Palatini formalism the lack of the kinetic terms in the action (2.23) leads to a single-field model. The study of the Palatini case is performed at tree level in [85]. The metric case at tree level is addressed in large number of studies, see e.g. [86–91]. The quantum corrections in the model are studied in [92–95] where it has been found that the coefficient of the R^2 term α is related to the ξ by $\alpha \sim \xi^2/(8\pi^2)$. A full slow roll study of the Higgs-Starobinsky inflation in the metric formalism at the tree-level has been performed in the article III.

Let us review Higgs-Starobinsky inflation in the metric formalism. The Jordan frame action for the model is obtained from the general action (2.23) by substituting the Higgs potential

$$\mathcal{L}_{\rm mat} = -\frac{1}{2} g^{\alpha\beta}\partial_\alpha h \partial_\beta h - \frac{\lambda}{4}(h^2 - \nu^2)^4, \tag{5.1}$$

and the extensions

$$q(h) = \xi h^2, \qquad f(\mathcal{R}) = \mathcal{R} + \alpha \mathcal{R}^2, \tag{5.2}$$

of the gravity sector therein. The negative values of α are excluded in order to avoid the saddle point around the EW vacuum. The substitutions (5.1) and (5.2) result in an Einstein frame action of the form

$$S = \int d^4x \sqrt{-g}\Big[\frac{1}{2}R - \frac{3}{4\Omega^4} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi - \\ \frac{3\xi h}{\Omega^4} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta h - \frac{\Omega^2 + 6\xi^2 h^2}{2\Omega^4} g^{\alpha\beta}\partial_\alpha h\partial_\beta h - \hat{V}(h,\phi)\Big], \tag{5.3}$$

with the potential

$$\hat{V}(h,\phi) = \frac{\lambda}{4}\frac{(h^2-\nu^2)^4}{\Omega^4} + \frac{1}{8\alpha}\frac{(\phi-1)^2}{\Omega^4}. \tag{5.4}$$

The slow roll regions for the model (5.3) that may lead to successful inflation are divided in two: the Starobinsky region, for which $h \ll 1$ and $\phi \gg 1$, and the region around the parabola $\phi \sim b+ch^2$. For positive ξ, the Starobinsky region corresponds to a hill in the h direction, causing the field to roll away from the EW vacuum. Only if the field is initially close enough to the line $h=0$ can it relax into the EW vacuum. Actually, this kind of solutions turn out to be the only possible solutions within the Starobinsky region.

The other successful slow roll region is a band around the parabola $\phi \sim b+ch^2$. The width of the band depends on the value of ξ. For small values $\xi \ll 1$ it becomes wide and connects with the Starobinsky region discussed above. Again, the only solutions compatible with the observations are the ones close to a limiting case, which now is the attractor parabola. This effective single field behaviour might be a consequence of the approximate scale invariance, as discussed in the final section.

An interesting feature of the Higgs-Starobinsky model will be that pure Higgs inflation seems to become impossible. This results from the asymmetry between the Higgs and the scalaron of the R^2 term: a constant scalaron can never be the solution to the equations of motion while the Higgs is dynamical (but the Higgs can be constant while the scalaron is dynamical). The Higgs inflation limit will be discussed in the final section of this chapter.

The analysis of Higgs-Starobinsky inflation is most practically performed in terms of frame covariant quantities. Let us first introduce the machinery for the frame covariant two-field inflation and the frame covariant observables and then discuss the successful slow roll regions in more detail.

5.2 Frame covariant approach for two-field inflation

The field space corresponding to the action (5.3) is not flat due to the existence of the kinetic cross terms and the field dependent coefficient of the usual kinetic terms. Rather, the field space is described by the metric

$$G_{AB} = \frac{1}{\Omega^4}\begin{pmatrix} 3/2 & 3\xi h \\ 3\xi h & \Omega^2 + 6\xi^2 h^2 \end{pmatrix}, \tag{5.5}$$

where the indices A, B refer to the field coordinates $\varphi^A = (\phi, h)$. The fact that the metric (5.5) is not transformable into the unit diagonal form by any field reparametrisation leads to several different choices of carrying out the analysis of the Higgs-Starobinsky inflation.

The conformal transformation followed by the field reparametrisation is called a frame transformation. The general result is that the physical predictions of the model considered should be frame-independent at least at the classical level. In order to ensure the frame independence of the observables and avoid certain confusion related to different parametrisations of the fields, the analysis is carried out in the frame covariant formalism [32, 57], as we have done.

Let us shortly review the main points. The goal of the analysis is to define the frame covariant slow roll parameters that ensure the frame invariance of the observables expanded in these parameters. First, one has to define the frame covariant equation of motions, which is achieved by using the general frame transformation rule to define the frame-covariant derivative and the frame covariant Hubble parameter. The transformation rule is

$$\nabla_C X^{A_1 A_2 \cdots A_p}_{B_1 B_2 \cdots B_q} = X^{A_1 A_2 \cdots A_p}_{B_1 B_2 \cdots B_q , C} + \Gamma^{A_1}_{CD} X^{D A_2 \cdots A_p}_{B_1 B_2 \cdots B_q} + \cdots - \Gamma^{D}_{CB_1} X^{A_1 A_2 \cdots A_p}_{D B_2 \cdots B_q} + \cdots \; . \tag{5.6}$$

where Γ^C_{AB} is the Levi-Civita connection of the field space with respect to the metric G_{AB}. Using this, the frame covariant field derivative with respect to any parameter λ is given by

$$\mathcal{D}_\lambda X^{A_1 A_2 \cdots A_p}_{B_1 B_2 \cdots B_q} = \frac{d\varphi^C}{d\lambda} \nabla_C X^{A_1 A_2 \cdots A_p}_{B_1 B_2 \cdots B_q}, \tag{5.7}$$

and the frame covariant Hubble parameter can be defined as

$$\mathcal{H} \equiv \frac{\mathcal{D}_t a}{a}. \tag{5.8}$$

Further, the frame covariant extension of the field equation in terms of (5.7) and (5.8) becomes

$$\mathcal{D}_t \mathcal{D}_t \varphi^A + 3\mathcal{H}(\mathcal{D}_t \varphi^A) + G^{AB} U_{,B} = 0, \tag{5.9}$$

which defines the geodesics in a curved manifold described by the metric (5.5) and allows for the definition of the slow roll parameters. The inflationary condition for the field takes the form $\mathcal{D}_t \mathcal{D}_t \varphi^A \ll \mathcal{H}(\mathcal{D}_t \varphi^A)$ that further gives the inflationary attractor

$$3\mathcal{H}(\mathcal{D}_t \varphi^A) + G^{AB} U_{,B} = 0. \tag{5.10}$$

For the slow roll parameters this implies the forms

$$\epsilon = \tfrac{1}{2} \tfrac{G^{AB} V_{,A} V_{,B}}{V^2}, \qquad \eta = G^{AB} \frac{V_{,A}}{V} \frac{\epsilon_{,B}}{\epsilon}, \tag{5.11}$$

which should reduce to the slow roll parameters defined in terms of the Hubble parameter and its derivatives.

Given the slow roll parameters (5.11) there is yet one thing to take into account before writing down the observables. As discussed in section (3.1), since in multifield inflation perturbations do not necessarily stay adiabatic or constant on superhorizon scales, isocurvature perturbations and non-gaussianity may be generated. This is due to the entropy flow between the curvature and isocurvature perturbations. Thus, the power spectra of the scalar and tensor perturbations, sourced by the curvature mode, are also affected by the isocurvature fraction (3.44), see equation (3.45). Using the local basis (σ, s) where the coordinates refer to the components parallel and perpendicular to the trajectory respectively (*i.e.* curvature and isocurvature modes) and the Sasaki-Mukhanov variables [96], the power spectra in our case become [32]

$$P_\mathcal{R} = \frac{\mathcal{H}^2}{8\pi^2 \bar{\epsilon}_\mathcal{H}}, \qquad P_\mathcal{T} = \frac{2\mathcal{H}^2}{\pi^2}, \tag{5.12}$$

where the frame-covariant Hubble slow roll parameters are defined as

$$\bar{\epsilon}_{\mathcal{H}} = -\frac{\mathcal{D}_t\mathcal{H}}{\mathcal{H}^2}, \qquad \bar{\eta}_{\mathcal{H}} = \frac{\mathcal{D}_t\bar{\epsilon}_{\mathcal{H}}}{\mathcal{H}\bar{\epsilon}_{\mathcal{H}}}. \tag{5.13}$$

Applying (5.12), (3.45) and $\epsilon_U \approx \epsilon_{\mathcal{H}}$ (and similarly for other SR parameteres) to the definitions of the observables (3.23), (3.24) and (3.27), the observables of the spectrum are found to be [32]

$$\begin{aligned} r \quad &= 16\epsilon\cos^2(\Theta), \\ n_R \quad &= 1 - 2\epsilon + \eta - \sin(2\Theta)\mathcal{D}_N T_{\mathcal{RS}}, \\ \alpha_R \quad &= -2\epsilon\eta - \eta\zeta - 2\cos(2\Theta)\sin^2(\Theta)(\mathcal{D}_N T_{\mathcal{RS}})^2 + \sin(2\Theta)\mathcal{D}_N\mathcal{D}_N T_{\mathcal{RS}}, \end{aligned} \tag{5.14}$$

while the dependence of the non-gaussianity parameter (5.15) on the number of efolds N is [32]

$$f_{NL} = \frac{5}{6}\frac{N^{,A}N^{,B}(\nabla_A\nabla_B N)}{(N_{,A}N^{,A})^2}. \tag{5.15}$$

5.3 Successful slow roll regions

Let us then consider the possible field trajectories and attractor solutions defined by the solutions to field equations (5.10). The possible slow roll regions are the Starobinsky region $h \ll 1$ and $\phi \gg 1$ and the effective single-field region around the parabola

$$\phi = 1 + b + ch^2, \tag{5.16}$$

with the constants

$$\begin{aligned} b \quad &= \quad -\frac{2\lambda\alpha}{12\lambda\alpha\xi + \xi^2(1+6\xi)} = \frac{c}{(c+d)(1-6d)}, \\ c \quad &= \quad \frac{2\lambda\alpha}{\xi}, \\ d \quad &= \quad \xi + \frac{1}{6}. \end{aligned} \tag{5.17}$$

Whether these regions are repellers or attractors is something that is controlled by the value of ξ: for $\xi < 0$ only the Starobinsky region can lead to viable inflation while for $\xi > 0$ both regions may be viable. In the latter case, the Starobinsky region is an attractor and in the former case a repeller. When it is a repeller, *i.e.* when $\xi < 0$, only the solution initially so close to the line $h = 0$ that the parameters are essentially equal to the exact Starobinsky case turns out to be possible. Instead, for $\xi > 0$, the attractor (5.16) solutions are favoured. The band around the attractor is wider for smaller ξ and becomes more narrow for larger ξ. If $\xi \ll 1$, the two slow roll regions are connected. In that case, a solution starting from the Starobinsky region and evolving towards the attractor parabola could in principle exist. However, the amount of isocurvature perturbation becomes observationally too large for such solutions thus ruling them out. The field trajectories corresponding to the different situations are illustrated in Figs. 1-3 in [3].

5.4 Limiting cases

Considering the frame covariant expressions (5.11) for the slow roll parameters, the limiting cases, the Starobinsky inflation and the Higgs inflation, can be considered in a unified manner. In the Starobinsky limit $h \ll 1$ ($\nu \ll 1$) one obtains

$$\epsilon = \frac{4}{3(\phi-1)^2} + \mathcal{O}(h^2), \tag{5.18}$$

$$\eta = \frac{8\varphi}{3(\phi-1)^2} + \mathcal{O}(h^2), \tag{5.19}$$

which equals the single-field result computed from the Starobinsky potential (3.77). Instead, similar deduction in the Higgs inflation limit $h \gg 1$ does not lead to results equal to the single-field case (3.68). In the limit $h \gg 1$ we get instead

$$\epsilon = \left(\frac{4}{3} + 8\xi\right) + \mathcal{O}\left(\frac{1}{h^2}\right), \tag{5.20}$$

$$\eta = \frac{4\xi(6\alpha\lambda - \xi - 6\xi^3)}{3\alpha\lambda} + \mathcal{O}\left(\frac{1}{h^2}\right). \tag{5.21}$$

This does not reduce to the Higgs inflation result, neither for the case $\phi = 1$ nor for $\alpha = 0$. Thus, pure Higgs inflation seems to become impossible when the $\mathcal{R}^2$ term is present in the action. However, when considering the observables in terms of the number of e-folds, the results may also be similar to the single-field Higgs inflation at the attractor solution of the field equations (5.10). The difference is that in such a case of effective Higgs inflation the field ϕ still affects the dynamics.

Let us finally discuss the fact that the solution to the equations of motion is so tightly bound on the attractor parabola at large field values. This could be a consequence of the approximate scale invariance at high energies where the term $\mathcal{R}/2$ can be ignored in the action. The lack of dimensional parameters then manifests as a scale invariance. The scale invariant transformations of the metric and the particle fields Φ are [77]

$$g_{\mu\nu}(x) \mapsto g_{\mu\nu}(\sigma x), \qquad \Phi(x) \mapsto \sigma^{d_\Phi}\Phi(x), \tag{5.22}$$

and the associated conserved current is [5]

$$\sqrt{-g}J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu g_{\alpha\beta})}\Delta g_{\alpha\beta} + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\phi, \tag{5.23}$$

where ϕ stands for the different fields. The symmetry following from current conservation, $\mathcal{D}_\mu J^\mu = 0$, then gives a constraint that removes one degree of freedom, leaving us with an effectively single field model at large field values. The study of the scale invariance in two-field models of inflation and its implication on the lack of isocurvature perturbations is discussed in [77, 97, 98].

Chapter 6

Conclusions and outlook

A large number of different phenomenological models of inflation are consistent with the observational data of the nearly scale invariant, highly Gaussian and adiabatic perturbations [38]. Then, models with a minimum amount of new assumptions or extensions of GR and SM should be worthy of attention [99]. Higgs inflation [14] is an example of such a minimalistic model that agrees with all the observations.

In this thesis, we have considered different aspects of Higgs inflation. The model and its predictions vary due to two different sources: quantum corrections, and the choice of gravitational degrees of freedom. For the latter, the metric and the Palatini formulations of GR result in different inflationary potentials and therefore in different predictions. The quantum corrections, instead, may lead to features such as a hilltop or an inflection point features in the inflationary potential which modify the predicted values of the observables from the tree-level case. From the total of two times two alternatives for the pure Higgs inflation, the thesis covers three [1, 2], the Palatini inflection point not yet having been studied. When the Higgs inflation model is extended to include also the Starobinsky term R^2, one obtains a two field inflationary model studied in [3].

In the papers [1–3] we have systematically studied the parameter space in order to find all the possible predictions of the model at hand. This includes finetuned parameter configurations $\xi, \Delta\lambda, \Delta y_t$ leading to spesific features in the Higgs potential. In general, succesful Higgs inflation requires non-zero positive values for the jump $\Delta\lambda$, which in the potential parametrises non-renormalisable physics. Solutions with $\Delta\lambda = 0$ are also possible with a low enough top quark mass [2]. As a general trend, configurations with a feature generally allows to obtain smaller values for the non-minimal coupling ξ and tensor-to-scalar ratio r than in the case when the potential is a simple plateau. In the Higgs Starobinsky model, the non-minimal coupling can be small $\xi \ll 1$ thus avoiding the unitarity problem. However, in that model we found that pure Higgs inflation becomes impossible due to the presence of R^2 term.

Signatures of Higgs inflation are testable by measurements of cosmological observables. The future

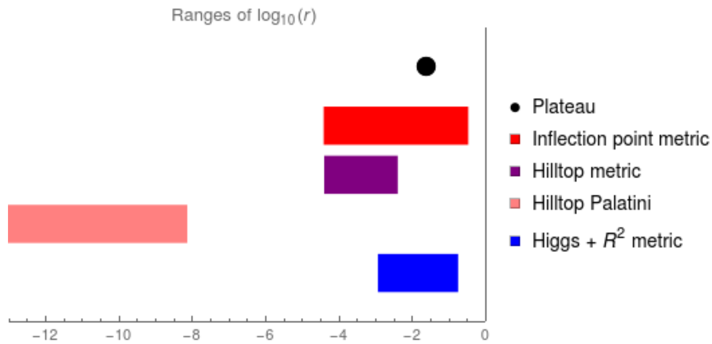

Figure 6.1: The tensor-to-scalar ratio in different variants of Higgs inflation based on [1–3]. The other observables are bounded on their observational ranges (3.39).

CMB experiments COrE[1], LiteBIRD[2] [100] ad PIXIE[3] [101] will be able to detect gravitational waves as small as $r \sim 10^{-3}$. Such a detection would possibly rule out the Palatini version of Higgs inflation and distinguish between the plateau, inflection point and hilltop inflation. In contrast, a new observational constraint $r < 10^{-3}$ could only rule out plateau Higgs inflation. The predictions for the tensor-to-scalar ratio in the cases studied in [1–3] are summarised in Fig. 6.1. In this figure, the range of r is restricted so that the spectral index n_s and its running α_s lie within their 95% confidence interval (3.39)[4]. Of course, independently of the value of r, the detection of a small enough running of the spectral index $\alpha_s \leq -0.01$ could rule out any Higgs inflation model. On the other hand, possible future observations [102] of considerable non-gaussianity and isocurvature could favour the Higgs-Starobinsky model over the single field models.

[1]http://www.core-mission.org/
[2]http://litebird.jp/eng/
[3] https://asd.gsfc.nasa.gov/pixie/
[4] For the detailed setups where the different ranges in the figure were obtained, see [1–3]. For instance, the reheating condition was only applied in [1] and the quantum corrections were not included in [3].

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